SEDIMENTATION

Sedimentation is the process of removing solid particles heavier than water by gravity settling. It is the oldest and most widely used unit operation in water and wastewater treatments. The terms sedimentation, settling, and clarification are used interchangeably. The unit sedimentation basin may also be referred to as a sedimentation tank, clarifier, settling basin, or settling tank.

In wastewater treatment, sedimentation is used to remove both inorganic and organic materials which are settleable in continuous-flow conditions. It removes grit, particulate matter in the primary settling tank, and chemical flocs from a chemical precipitation unit. Sedimentation is also used for solids concentration in sludge thickeners.

Based on the solids concentration, and the tendency of particle interaction, there are four types of settling which may occur in wastewater settling operations. The four categories are discrete, flocculant, hindered (also called zone), and compression settlings. They are also known as types 1, 2, 3, and 4 sedimentation, respectively. Some discussion of sedimentation is covered in Chapter 1.6. The following describes each type of settling.

Discrete Particle Sedimentation (Type 1)

The plain sedimentation of a discrete spherical particle, described by Newton's law, can be applied to grit removal in grit chambers and sedimentation tanks. The terminal settling velocity is determined as (also in Chapter 1.6, Eq. (6.60))

$$v_s = \left[\frac{4g(\rho_s - \rho)d}{3C_D\rho}\right]^{1/2} \tag{7.33}$$

where v_s = terminal settling velocity, m/s or ft/s
ρ_s = mass density of particle, kg/m^3 or lb/ft^3
ρ = mass density of fluid, kg/m^3 or lb/ft^3
g = acceleration due to gravitation, 9.81 m/s^2 or 32.2 ft/s^2
d = diameter of particle, mm or in
C_D = dimensionless drag coefficient

The drag coefficient C_D is not constant. It varies with the Reynolds number and the shape of the particle. The Reynolds number $\mathbf{R} = vd\rho/\mu$, where μ is the absolute viscosity of the fluid, and the other terms are defined as above.

C_D varies with the effective resistance area per unit volume and shape of the particle. The relationship between **R** and C_D is as follows

$$1 > \mathbf{R}: \quad C_D = \frac{24}{\mathbf{R}} = \frac{24\mu}{v\rho d} \tag{7.34}$$

$$1 < \mathbf{R} < 1000: \quad C_D = \frac{24}{\mathbf{R}} + \frac{3}{\mathbf{R}^{0.5}} + 0.34 \tag{7.35}$$

$$\text{or} = \frac{18.5}{\mathbf{R}^{0.5}} \tag{7.36}$$

$$\mathbf{R} > 1000: \quad C_D = 0.34 \text{ to } 0.40 \tag{7.37}$$

For small **R** (< 1 or 2) with laminar flow. Eq. (7.34) is applied. Eq. (7.35) or (7.36) is applicable for **R** up to 1000, which includes all situations of water and wastewater treatment processes. For fully developed turbulent settling use $C_D = 0.34$ to 0.40 (Eq. (7.37)).

When the Reynolds number is less than 1, substitution of Eq. (7.34) for C_D in Eq. (7.33) yields Stokes' law (Eq. (6.63))

$$v_s = \frac{g(\rho_s - \rho)d^2}{18\mu} \tag{7.38}$$

Discrete particle settling refers to type 1 sedimentation. Under quiescent conditions, suspended particles in water or wastewater exhibit a natural tendency to agglomerate, or the addition of coagulant chemicals promotes flocculation. The phenomenon is called flocculation–sedimentation or type 2 sedimentation. For flocculated particles the principles of settling are the same as for a discrete particle, but settling merely occurs at a faster rate.

Scour

The horizontal velocity in grit chambers or in sedimentation tanks must be controlled to a value less than what would carry the particles in traction along the bottom. The horizontal velocity of fluid flow just sufficient to create scour is described as (Camp 1946)

$$V = \left[\frac{8\beta(s-1)gd}{f}\right]^{1/2} \tag{7.39}$$

where V = horizontal velocity, m/s
β = constant for the type of scoured particles
= 0.04 for unigranular material
= 0.06 for sticky interlocking material
s = specific gravity of particle
g = acceleration due to gravity, 9.81 m/s^2
d = diameter of particle, m
f = Darcy–Weisbach friction factor, 0.02–0.03

The f values are a function of the Reynolds number and surface characteristics of the settled solids. The horizontal velocity in most sedimentation tanks is well below that which would cause scour. In grit chambers, scour is an important factor for design.

EXAMPLE: Determine the surface overflow rate and horizontal velocity of a grit chamber to remove the grit without removing organic material. Assume that grit particles have a diameter of 0.2 mm (0.01 in) and a specific gravity of 2.65 (sand, silt, and clay); and organic material has the same diameter and a specific gravity of 1.20. Assume $C_D = 10$.

Solution:

Step 1. Compute the terminal settling velocity, using Eq. (7.33)

$$C_D = 10$$

$$d = 0.2 \text{ mm} = 0.02 \text{ cm}$$

$$v_s = \left[\frac{4g(\rho_s - \rho)d}{3C_D\rho}\right]^{1/2}$$

$$= \left[\frac{4 \times 981 \times (2.65 - 1) \times 0.02}{3 \times 10 \times 1}\right]^{1/2}$$

$$= 2.08 \text{ (cm/s)}$$

Note: This will be the surface overflow rate to settle grit, not organic matter.

Step 2. Compute the horizontal velocity (V_1) just sufficient to cause the grit particles to scour Use $\beta = 0.06$ and $f = 0.03$. Using Eq. (7.39)

$$V_1 = \left[\frac{8\beta(s - 1)gd}{f}\right]^{1/2}$$

$$= \left[\frac{8 \times 0.06 \times (2.65 - 1) \times 981 \times 0.02}{0.03}\right]^{1/2}$$

$$= 22.8 \text{ (cm/s)}$$

Step 3. Compute the scouring velocity V_2 for organic material, using Eq. (7.39)

$$V_2 = \left[\frac{8 \times 0.06 \times (1.20 - 1) \times 981 \times 0.02}{0.03}\right]^{1/2}$$

$$= 7.9 \text{ (cm/s)}$$

Note: The grit chamber is designed to have a surface overflow rate (settling velocity) of 2.1 cm/s and a horizontal velocity less than 22.8 cm/s but greater than 7.9 cm/s. Under these conditions, the grit will be removed and organic matter will not. If the horizontal velocity is close to the scour velocity, the grit will be reasonably clean.

Sedimentation Tank (Basin) Configuration

Sedimentation tanks can be rectangular, square, or circular. Imhoff tanks perform the dual function of settling and aerobic treatment with two-story chambers; however, the Imhoff tank is old technology and is no longer allowed in the developed countries.

For a continuous flow sedimentation tank, the shape can be either rectangular or circular. Camp (1953) divided the ideal sedimentation tank into four zones which affect settling, namely the inlet zone, theoretical effective settling zone, sludge zone (beneath the settling zone), and outlet zone (Fig. 7.6). The inlet and outlet condition and tank geometry influence short circuiting, which can be minimized in narrow rectangular horizontal flow basins. Short circuiting is a common problem in circular radial flow clarifiers.

Figure 7.6 illustrates an ideal rectangular continuous horizontal flow settling tank. The inlet zone uniformly distributes wastewater flows and solids over the cross-sectional area of the tank in such a manner that flow through the settling zone follows horizontal paths to prevent short circuiting. In the settling zone, a uniform concentration of particles settles at terminal settling velocity to the sludge zone at the bottom of the tank. In the real world, there is no theoretical effective settling zone. Particle settling vectors are difficult to predict. However, it is usually assumed that the flow of wastewater through the settling zone is steady and that the concentration of each sized particle is uniform throughout the cross section normal to the flow direction. The sludge zone is a region for storing the settled sediments below the settling zone. This zone may be neglected for practical purposes, if mechanical equipment continually removes the sediment. In the outlet zone, the supernatant (clarified effluent) is collected through an outlet weir and discharged to further treatment units or to the outfall.

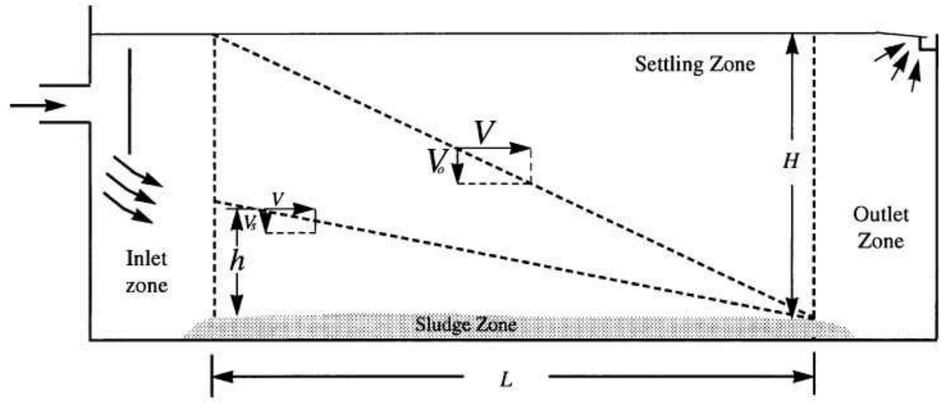

FIGURE 7.6 Sketch of the discrete particle settling in an ideal settling tank.

In the design of clarifiers, a particle terminal velocity V_0 is used as a design overflow settling velocity, which is the settling velocity of the particle which will settle through the total effective depth H of the tank in the theoretical detention time. All particles that have a terminal velocity (V_s) equal to or greater than V_0 will be removed. The surface overflow rate of wastewater is (Stoke's law)

$$V_0 = Q/A = Q/WL \tag{7.40}$$

$$= \frac{g(\rho_s - \rho)d^2}{18\mu} \tag{7.41}$$

where Q = flow, m^3/d or gal/d
A = surface area of the settling zone, m^2 or ft^2
V_0 = overflow rate or surface loading rate, $m^3/(m^2 \cdot d)$ or $gal/(ft^2 \cdot d)$
W, L = width and length of the tank, m or ft

This is called type 1 settling. Flow capacity is independent of the depth of a clarifier. Basin depth H is a product of the design overflow velocity and detention time t

$$H = V_0 t \tag{7.42}$$

The flow through velocity V_f is

$$V_f = Q/HW \tag{7.43}$$

where H is the depth of the settling zone. The retention time t is

$$t = \text{Volume}/Q \tag{7.44}$$

The removal ratio r (or fraction of removal) of particles having a settling velocity equal to V_s will be h/H. Since depth equals the product of the settling velocity and retention time t (Fig. 7.6)

$$r = \frac{h}{H} = \frac{V_s t}{V_0 t} = \frac{V_s}{V_0} \tag{7.45}$$

where r is the fraction of the particles with settling velocity V_s that are removed. This means that in a horizontal flow particles with settling velocity V_s less than V_0 will also be removed if they enter the settling zone at a depth less than H.

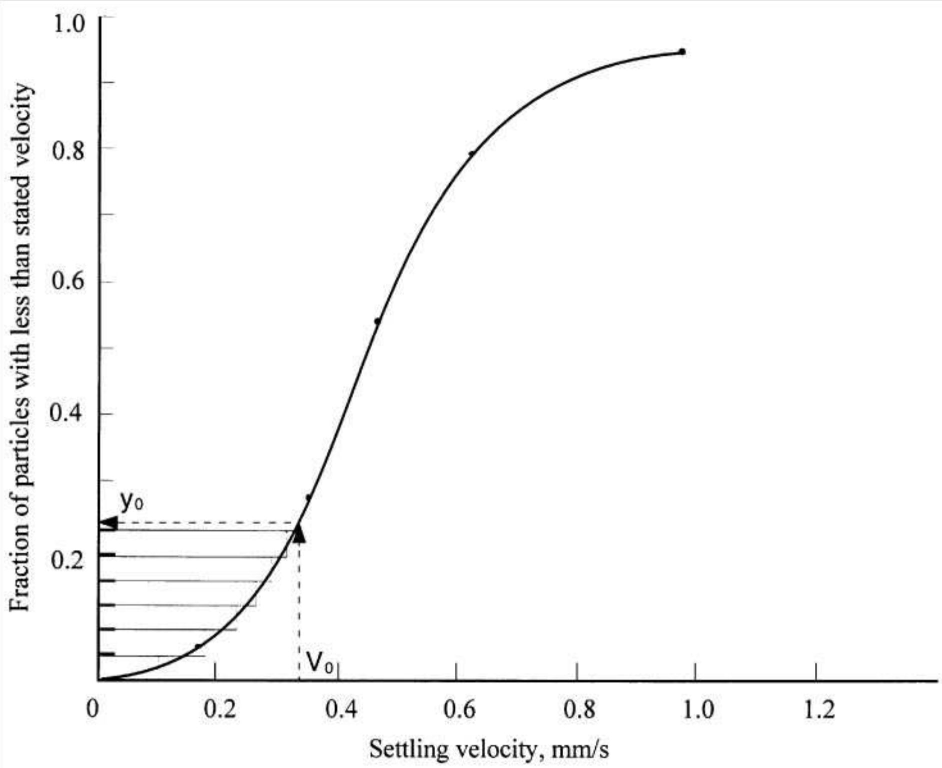

FIGURE 7.7 Cumulative particles removal versus settling velocity curve.

The settling velocity distribution for a suspension sample can be determined from a column settling test. The data obtained from the test can be used to construct a cumulative settling velocity frequency distribution curve, as shown in Fig. 7.7.

For a given clarification flow rate Q, only those particles having settling velocity $\geq V_0$ ($= Q/A$) will be completely removed. Let y_0 represent the portion of particles with a settling velocity $< V_0$; then the percentage removed will be $1 - y_0$. Also, for each size particle with $V_s < V_0$ its proportion of removal, expressed as Eq. (7.45), is equal to $r = V_s/V_0$. When considering various particle sizes in this group, the percentage of removal is

$$\int_0^{y_0} \frac{V_s}{V_0} dy$$

The overall fraction of particles removed, F, would be

$$F = (1 - y_0) + \frac{1}{V_0}\int_0^{y_0} V_s\, dy \tag{7.46}$$

Approximation:

$$F = 1 - y_0 + \frac{V_0 + V_1}{2V_0}(y_0 - y_1) + \frac{V_1 + V_2}{2V_0}(y_1 - y_2) + \cdots + \frac{V_i + V_{i+1}}{2V_0}(y_1 - y_{i+1})$$

$$F = 1 - y_0 + \frac{1}{V_0}\sum V\Delta y \tag{7.47}$$

where y_0 = fraction of particles by weight with $V_s \geq V_0$
i = ith particle

EXAMPLE (type 1): A clarifier is designed to have a surface overflow rate of 28.53 $m^3/(m^2 \cdot d)$ (700 gal/($ft^2 \cdot d$)). Estimate the overall removal with the settling analysis data and particle size distribution in cols. 1 and 2 of Table 7.8. The wastewater temperature is 15°C and the specific gravity of the particles is 1.20.

Solution:

Step 1. Determine settling velocities of particles by Stokes' law, Eq. (7.41)
From Table 5.1a, at 15°C

$$\mu = 0.00113 \text{ N} \cdot \text{s/m}^2 = 0.00113 \text{ kg/(s} \cdot \text{m)}$$

$$\rho = 0.9990$$

$$V = \frac{g(\rho_s - \rho)d^2}{18\mu}$$

$$= \frac{9.81 \text{ m/s}^2(1200 - 999) \text{ kg/m}^3 \times d^2}{18 \times 0.00113 \text{ kg/(s} \cdot \text{m)}}$$

$$= 96{,}942d^2 \text{ m/s}$$

where d is in m

Step 2. Calculate V for each particle size (col. 3 of Table 7.9)
For d = 0.1 mm = 0.0001 m

$$V = 96{,}942\ (0.0001)^2$$
$$= 0.000969 \text{ m/s}$$
$$= 0.968 \text{ mm/s}$$

Similarly, calculate the settling velocities for other particle sizes.

Step 3. Construct the settling velocities versus cumulative distribution curve shown in Fig. 7.7.

Step 4. Calculate designed settling velocity V_0

$$V_0 = 28.53 \text{ m/d}$$
$$= 28{,}530 \text{ mm/d} \times 1 \text{ day}/86{,}400 \text{ s}$$
$$= 0.33 \text{ mm/s}$$

TABLE 7.9 Results of Settling Analysis Test and Estimation of Overall Solids Removal

Particle size mm	Weight fraction < size, %	Settling velocity V, mm/s
0.10	12	0.968
0.08	18	0.620
0.07	35	0.475
0.06	72	0.349
0.05	86	0.242
0.04	94	0.155
0.02	99	0.039
0.01	100	0.010

Note: All particles with settling velocities greater than 0.33 mm/s (700 gal/(d · ft^2)) will be removed.

Step 5. Find the fraction $(1 - y_0)$
From Fig. 7.7 we read $y_0 = 0.25$ at $V_0 = 0.33$ mm/s

then $$1 - y_0 = 1 - 0.25 = 0.75$$

Step 6. Graphical determination of $\Sigma V\Delta y$

Referring to Figure 7.7

Δy	0.04	0.04	0.04	0.04	0.04	0.04	0.01
V	0.09	0.17	0.23	0.26	0.28	0.31	0.33
$V\Delta y$	0.0036	0.0068	0.0092	0.0104	0.0112	0.0124	0.0033

$$\sum V\Delta y = 0.0569$$

Step 7. Determine overall removal R
Using Eq. (7.47)

$$\begin{aligned} F &= (1 - y_0) + \frac{1}{V_0}\sum V\Delta y \\ &= 0.75 + 0.0569/0.33 \\ &= 0.92 \\ &= 92 \text{ percent} \end{aligned}$$

Flocculant Settling (Type 2)

In practice, the actual settling performance cannot be adequately predicted because of unrealistic assumptions on ideal discrete particle settling. Under quiescent conditions, suspended particles in water or wastewater exhibit a natural tendency to agglomerate. Also, suspended solids in wastewater are not discrete particles and vary more than light and small particles, as they contact and agglomerate and grow in size. As coalescence of flocculation occurs, including chemical coagulation and biological flocs, the mass of the particles increases and they settle faster. This phenomenon is called flocculant or type 2 sedimentation.

The flocculation process increases removal efficiency but it cannot be adequately expressed by equations. Settling-column analysis is usually used to determine the settling characteristics of flocculated particles. A column can be of any diameter and equal in length to the proposed clarifier. Satisfactory results can be achieved with 15 cm (6 in) diameter plastic tube 3 m (10 ft) in height (Metcalf and Eddy, Inc. 1991). Sampling ports are uniformly spaced (45 to 60 cm or 1.5 to 2 ft) along the length of the column. The test suspension is placed in the settle-column and allowed to settle in a quiescent manner. The initial suspended solids concentration is measured. Samples are withdrawn from the sampling ports at various selected time intervals from different depths. Analyses of SS are performed for each sample, and the data used to calculate the percentage of removal is plotted as a number against time and depth. Between the plotted points, curves of equal percent removal are drawn. The results of settling-column analyses are presented in Fig. 7.8. Use of the curves in Fig. 7.8 is illustrated in the following example.

EXAMPLE: Using the settling curves of Fig. 7.8, determine the overall removal of solids in a sedimentation basin (type 2 flocculant settling) with a depth equal to the test cylinder and at a detention time of 20 min. The total depth is 2.5 m.

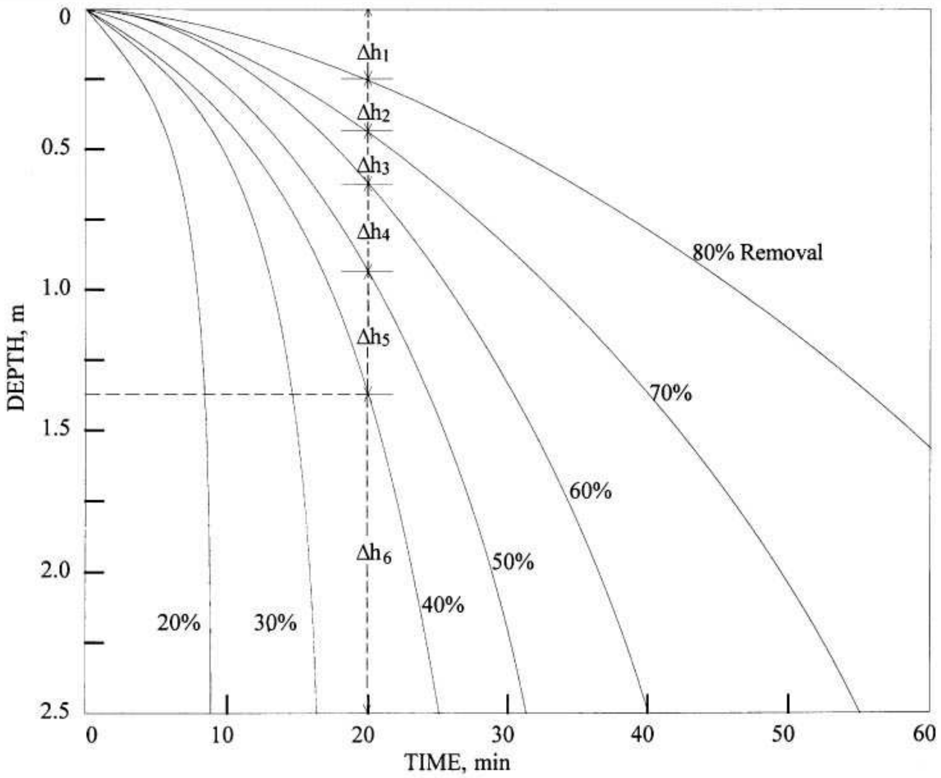

FIGURE 7.8 Settling characteristics for flocculent particles.

Solution:

Step 1. From Fig. 7.8, 40 percent of the particles will have a settling velocity of 0.1 m/min (2.5 m/25 min)

At $t = 20$ min, the volume of the test cylinder within Δh_6 has 40% removal

Step 2. Determine percent removal of each volume of the tank
In the volume of the tank corresponding to Δh_5 between 50 and 40 percent removal will occur. Similarly, in the tank volume corresponding to Δh_4 between 60 and 50 percent will be removed. In like fashion, this is applied to other tank volumes.

Step 3. Calculate the overall removal
Since $1/h = 1/2.5 = 0.4$

$\Delta h_1 = 0.23$ m

$\Delta h_2 = 0.14$ m

$\Delta h_3 = 0.20$ m

$\Delta h_4 = 0.32$ m

$\Delta h_5 = 0.50$ m

$$F = 40 + \frac{\Delta h_5}{h}\left(\frac{40 + 50}{2}\right) + \frac{\Delta h_4}{h}\left(\frac{50 + 60}{2}\right) + \frac{\Delta h_3}{h}\left(\frac{60 + 70}{2}\right)$$

$$+ \frac{\Delta h_2}{h}\left(\frac{70 + 80}{2}\right) + \frac{\Delta h_1}{h}\left(\frac{80 + 100}{2}\right)$$

$$= 40 + 0.4(0.5 \times 45 + 0.32 \times 55 + 0.20 \times 65 + 0.14 \times 75 + 0.23 \times 90)$$

$$= 73.7\% \text{ removal}$$

Hindered Sedimentation (Type 3)

In systems with high concentrations of suspended solids, the velocity fields of closely spaced particles are obstructed, causing an upward displacement of the fluid and hindered or zone settling (type 3) and compression settling (type 4). In addition, discrete (free) settling (type 1) and flocculant settling (type 2) occur. This settling phenomenon of concentrated suspensions (such as activated sludge) is illustrated in a graduated cylinder, as shown in Fig. 7.9.

Hindered (zone) settling occurs in sludge thickeners and at the bottom of a secondary clarifier in biological treatment processes. The velocity of hindered settling is estimated by (Steel and McGhee 1979)

$$v_h/v = (1 - C_v)^{4.65} \tag{7.48}$$

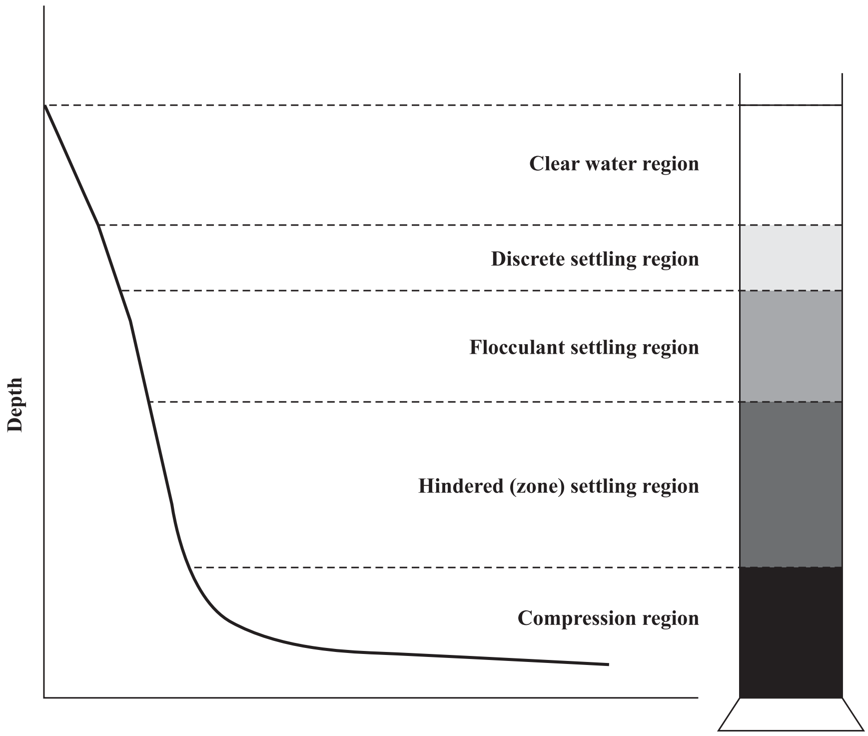

FIGURE 7.9 Schematic drawing of settling regions for concentrated suspensions.

where υ_h = hindered settling velocity, m/s or ft/s
υ = free settling velocity, calculated by Eq. (7.33) or (7.38)
C_v = volume of particles divided by the volume of the suspension

Eq. (7.48) is valid for Reynolds numbers less than 0.2, which is generally the situation in hindered settling.

A typical curve of interface height versus time for activated sludge is shown in Fig. 7.10. From A to B, there is a hindered settling of the particles and this is called liquid interface. From B to C there is a deceleration marking the transition from hindered settling into the compression zone. From C to D there is a compression zone where settling depends on compression of the sludge blanket.

The system design for handling concentrated suspensions for hindered settling must consider three factors: (1) the area needed for discrete settling of particles at the top of the clarifier; (2) the area needed for thickening (settling of the interface between the discrete and hindered settling zones); and (3) the rate of sludge withdrawal. The settling rate of the interface is usually the controlling factor.

Column settling tests, as previously described, can be used to determine the area needed for hindered settling. The height of the interface is plotted against time, as shown in Fig. 7.10. The area needed for clarification is

$$A = Q/\upsilon_s \tag{7.49}$$

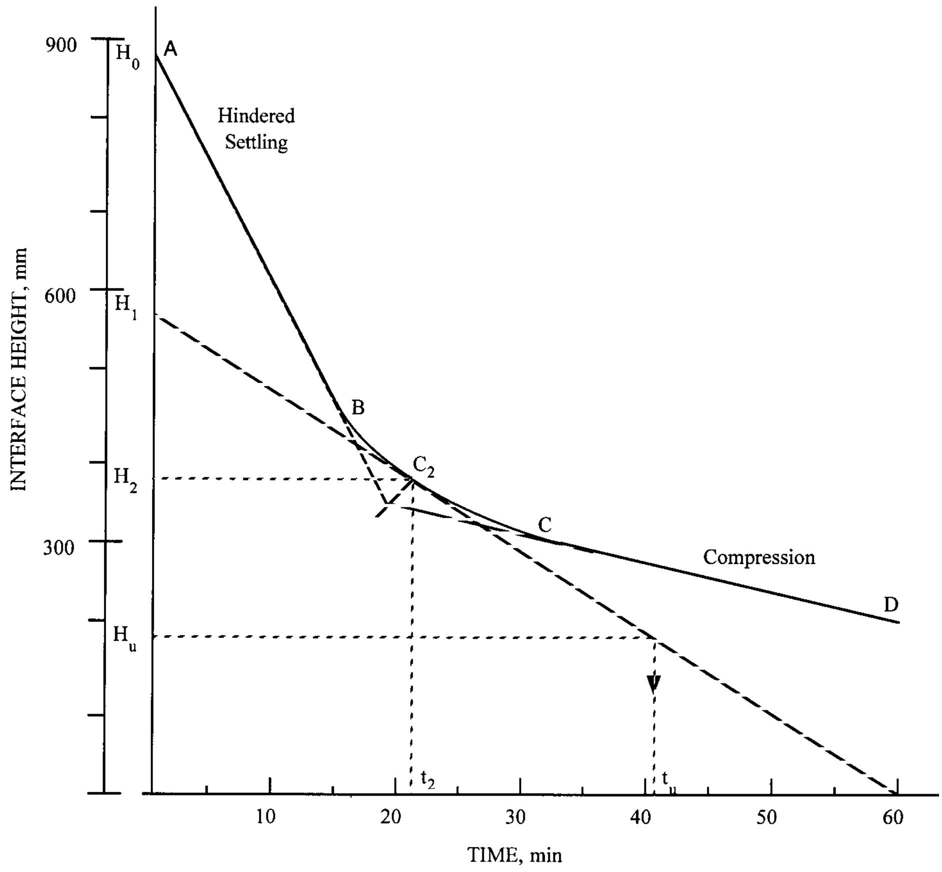

FIGURE 7.10 Graphic analysis of interface.

where A = surface area of the settling zone, m^2 or ft^2
Q = overflow rate, m^3/s or gal/min
υ_s = subsidence rate in the zone of hindering settling, mm/s or in/s

A value of υ_s is determined from batch settling column test data by computing the slope of the hindered settling portion of the interface height versus time curve (Fig. 7.10). The area needed for thickening is obtained from the batch settling test of a thick suspension. The critical area required for adequate thickening is (Rich 1961)

$$A = \frac{Qt_u}{H_0} \tag{7.50}$$

where A = area needed for sludge thickening, m^2 or ft^2
Q = flow into settling tank, m^3/s or ft^3/s
t_u = time to reach a desired underflow or solids concentration, s
H_0 = depth of the settling column (initial interface height), m or ft

From Fig. 7.10, the critical concentration (C_2) is determined by extending the tangent from the hindered and compression settling lines to their point of intersection and bisecting the angle formed. The bisector intersects the subsidence curve at C_2 which is the critical concentration. The critical concentration controls the sludge-handling capacity of the tank at a height of H_2.

A tangent is drawn to the subsidence curve at C_2 and the intersection of this tangent with depth H_u, required for the desired underflow (or solids concentration C_u), will yield the required retention time t_u. Since the total weight of solids in the system must remain constant, i.e. $C_0H_0A = C_uH_uA$, the height H_u of the particle–liquid interface at the underflow desired concentration C_u is

$$H_u = \frac{C_0H_0}{C_u} \tag{7.51}$$

The time t_u can be determined as:
draw a horizontal line through H_u and draw a tangent to the subsidence settling curve at C_2. Draw a vertical line from the point of intersection of the two lines drawn above to the time axis to find the value of t_u. With this value of t_u, the area needed for thickening can be calculated using Eq. (7.50). The area required for clarification is then determined. The larger of the two calculated areas from Eqs. (7.40) and (7.50) is the controlling factor for design.

EXAMPLE: The batch-settling curve shown in Fig. 7.10 is obtained for an activated sludge with an initial solids concentration C_0 of 3600 mg/L. The initial height of the interface in the settling column is 900 mm. This continuous inflow to the unit is 380 m^3/d (0.10 Mgal/d). Determine the surface area required to yield a thickened sludge of 1.8 percent by weight. Also determine solids and hydraulic loading rate.

Solution:

Step 1. Calculate H_u by Eq. (7.51)

$$C_u = 1.8\% = 18{,}000 \text{ mg/L}$$

$$H_u = \frac{C_0H_0}{C_u} = \frac{3600 \text{ mg/L} \times 900 \text{ mm}}{18{,}000 \text{ mg/L}}$$

$$= 180 \text{ mm}$$

Step 2. Determine t_u
Using the method described above to find the value of t_u

$$t_u = 41 \text{ min} = 41 \text{ min}/1440 \text{ min/d}$$

$$= 0.0285 \text{ day}$$

Step 3. Calculate the area required for the thickening, using Eq. (7.50)

$$A = \frac{Qt_u}{H_0} = \frac{380 \text{ m}^3/\text{d} \times 0.0285 \text{ d}}{0.90 \text{ m}}$$
$$= 12.02 \text{ m}^2$$
$$= 129 \text{ ft}^2$$

Step 4. Calculate the subsidence velocity v_s in the hindered settling portion of the curve
In 10 min

$$v_s = \frac{(900 - 617) \text{ mm}}{10 \text{ min} \times 60 \text{ s/min}}$$
$$= 0.47 \text{ mm/s}$$
$$= 40.6 \text{ m/d}$$

Step 5. Calculate the area required for clarification
Using Eq. (7.49)

$$A = Q/v_s = 380 \text{ m}^3/\text{d} \div 40.6 \text{ m/d}$$
$$= 9.36 \text{ m}^2$$

Step 6. Determine the controlling area
From comparison of areas calculated from Steps 3 and 5, the larger area is the controlling area. Thus the controlling area is the thickening area of 12.02 m^2 (129 ft^2)

Step 7. Calculate the solids loading

$$C_0 = 3600 \text{ mg/L} = 3600 \text{ g/m}^3 = 3.6 \text{ kg/m}^3$$
$$\text{Solids weight} = QC_0 = 380 \text{ m}^3/\text{d} \times 3.6 \text{ kg/m}^3$$
$$= 1368 \text{ kg/d}$$
$$= 3016 \text{ lb/d}$$

$$\text{Solids loading rate} = 1368 \text{ kg/d} \div 12.02 \text{ m}^2$$
$$= 114 \text{ kg/(m}^2 \cdot \text{d)}$$
$$= 23.3 \text{ lb/(ft}^2 \cdot \text{d)}$$

Step 8. Determine the hydraulic (overflow) loading rate

$$\text{Hydraulic loading rate} = 380 \text{ m}^3/\text{d} \div 12.02 \text{ m}^2$$
$$= 31.6 \text{ m}^3/(\text{m}^2 \cdot \text{d}) = 31.6 \text{ m/d}$$
$$\text{or} = 100{,}000 \text{ gal/d} \div 129 \text{ ft}^2$$
$$= 776 \text{ gal/(ft}^2 \cdot \text{d)}$$

Compression Settling (Type 4)

When the concentration of particles is high enough to bring the particles into physical contact with each other, compression settling will occur. Consolidation of sediment at the bottom of the clarifier is extremely slow. The rate of settlement decreases with time due to increased resistance to flow of the fluid.

The volume needed for the sludge in the compression region (thickening) can also be estimated by settling tests. The rate of consolidation in this region has been found to be approximately proportional to the difference in sludge height H at time t and the final sludge height H_∞ obtained after a long period of time, perhaps 1 day. It is expressed as (Coulson and Richardson 1955)

$$\frac{dH}{dt} = i(H - H_\infty) \tag{7.52}$$

where H = sludge height at time t
i = constant for a given suspension
H_∞ = final sludge height

Integrating Eq. (7.52) between the limits of sludge height H_t at time t and H_1 at time t_1, the resulting expression is

$$H_t - H_\infty = (H_1 - H_\infty)e^{-i(t-t_1)} \tag{7.53}$$

or

$$i(t - t_1) = \ln(H_t - H_\infty) - \ln(H_1 - H_\infty) \tag{7.54}$$

A plot of ln $[(H_t - H_\infty) - \ln(H_1 - \ln H_\infty)]$ versus $(t - t_1)$ is a straight line having the slope $-i$. The final sludge height H_∞ depends on the liquid surface film which adheres to the particles.

It has been found that gentle stirring serves to compact sludge in the compression region by breaking up the floc and permitting water to escape. The use of mechanical rakes with 4 to 5 revolutions per hour will serve this purpose. Dick and Ewing (1967) reported that gentle stirring also helped to improve settling in the hindered settling region.